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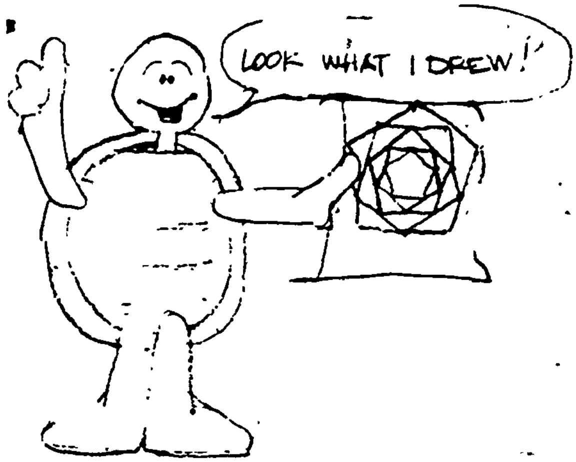
ABSTRACT

This document is the seventh of seven units developed by the Math Network Curriculum Project. Each unit, designed to be a 2-week module, is a teacher's guide which includes detailed directions along with the courseware and software needed. Teacher intervention in the non-computer activities that begin each unit is required, and the consistent use of small-group instruction makes the units usable in a standard classroom if two microcomputers are present. The Turtle Symmetry Unit helps students to learn to identify and draw designs by employing the principles of mirror symmetry and rotational symmetry. They also study and create "families" of designs--sets of designs which have common characteristics yet differ in patterned ways. This unit requires class use of the Turtleworks computer program (developed by Bill Finzer) and included with the teacher's guide to Turtle Geometry. It was developed for use on a Commodore PET Computer with at least 16K of RAM using 4.0 BASIC. (MNS)



Turtle Symmetry

Teacher's Guide

Math Network Curriculum Project

San Francisco State University

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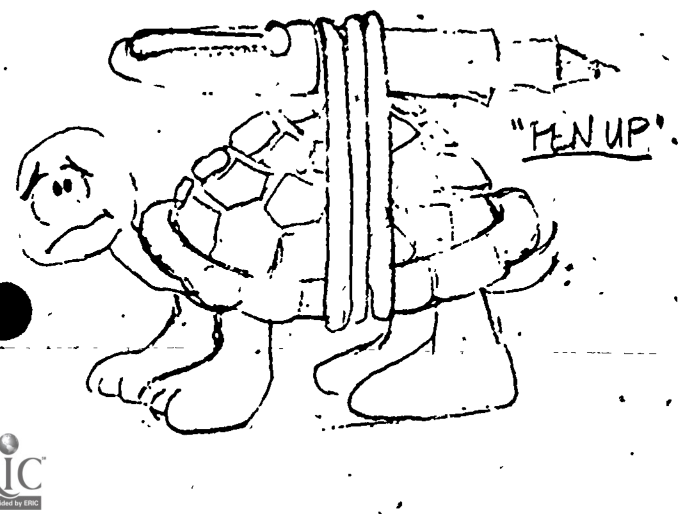

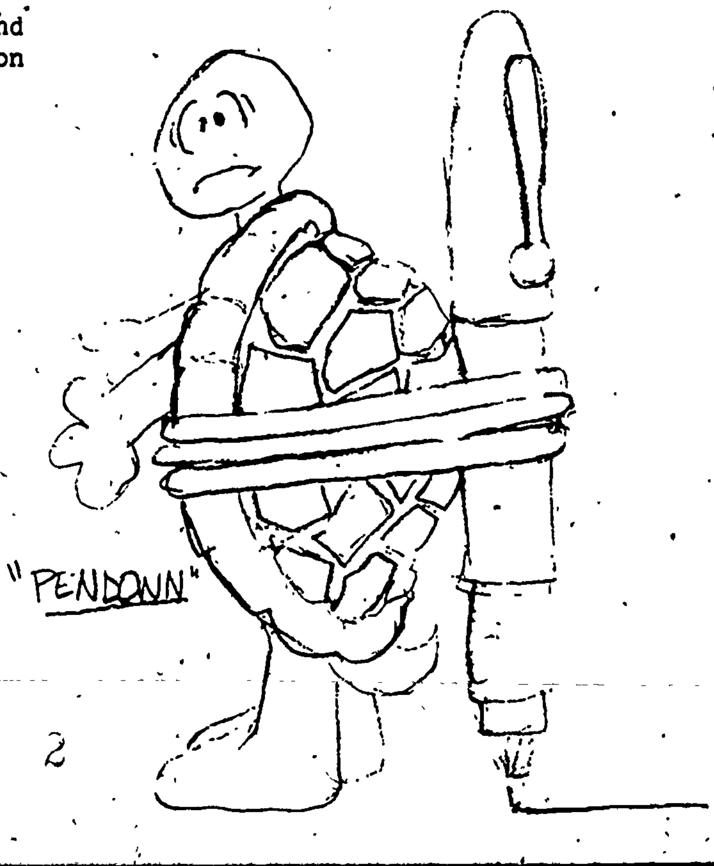

TURTLE SYMMETRY

TEACHER'S GUIDE

Math Network Curriculum Project

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Overview

What Is Symmetry About?

We encounter symmetry in the world beginning from a very young age, perhaps when we notice the way our right and left hands match palm to palm, or when we see a person standing in front of a mirror. From this beginning, our fascination grows. We play at being a copy-cat, 'mirroring' another's words or actions; we cut snowflakes from multiply-folded pieces of paper; we gaze through kaleidoscopes, stare at whorls of a rose, or a whirlpool; and we try our own hand at creating drawings with different kinds of symmetry.

An understanding of different kinds of symmetry leads to a deeper understanding of our world. The sociologist talks about symmetric relationships among people; the biologist classifies plants partly on the basis of their symmetries; the chemist uses the symmetries of molecules to understand their reactions; and the physicist explains the very foundations of the structure of matter using the properties of symmetry in time and space.

Mathematics provides a particularly simple, yet fertile, ground for coming to grips with the ideas of symmetry. In particular, in mathematics, we have a language for talking about these ideas.[1, 2]

What Do Turtles Have To Do With Symmetry?

Having already worked through 'Turtle Geometry,' which is a prerequisite for this unit, you and your students have met some of the ways in which turtles can bring symmetry to life. It is difficult, without a computer, to create symmetric designs. Drawing by hand is tedious and difficult. Tools such as mirrors and tiles are invaluable aides, but still do not allow for free-flowing expression of symmetry ideas. A multitude of turtles moving by command on the computer screen can quickly and easily create symmetric designs of infinite variety and great beauty. The Turtletalk language provides a medium for expressing symmetry ideas. Because this language addresses the turtles in terms of distances and angles on the screen, students acquire a quantitative, as well as qualitative, understanding of symmetry.

[1]Stevens, Peter, Handbook of Regular Patterns, An Introduction to Symmetry in Two Dimensions, MIT Press, 1980. Contains a clear exposition of the different kinds of symmetry in two dimensions and a multitude of examples from all through history and all over the world.

[2]Kim, Scott, Inversions, Byte Books, 1981. A book that will delight all! Wonderful examples of ways to bring symmetry into calligraphy.

How to Use This Guide

Turtle Geometry as a Prerequisite

The work in this unit presumes that your students have already completed the Turtle Geometry unit. Your students should be fluent in Turtletalk, and understand about angles, distances, and polygons. No mechanism is presented here for introducing these fundamental concepts.

Purpose of This Guide

This guide outlines activities that will introduce symmetry concepts to your students. In this way the computer functions as one of several learning tools in your classroom. The guide is also not meant to be a treatise on how to use computers, nor on symmetry. If you have felt successful in teaching Turtle Geometry, then you have dealt with many of the problems of using computers in your classroom. If you also have met symmetry concepts in other contexts, then you should be quite comfortable with teaching the material presented here. **In general, however, we assume that you are being introduced to these ideas in some course for teachers.** This guide is meant to serve as a reference for you in the classroom.

What You and Your Students Will Do

As in the Turtle Geometry unit, most of the classroom time in this unit will be taken up with small group work. Groups of three work well around a computer, while four seems to be a good size when the computer is not involved. You will probably want to choose a single group size for both situations. The size you found to work best in the Turtle Geometry unit will also work well in this unit.

In each activity, your task consists of four parts: preparing materials, introducing an activity, making sure that everything runs smoothly during the activity, and conducting a summary discussion.

Summary of Activities and Materials

Total Time Required: about 2 weeks
Total Whole Class Time: from 4 to 5 class periods
Independent Computer Time: 60 minutes per group of 3

1. **Alphabet Symmetry**
   Format - Whole class.
   Time - 1 period
   Materials
   Large cutout letters
   Overhead projector
   Turtle-tractor
   Greek Letters Symmetry Worksheet copies

   The terms **mirror symmetry** and **rotational symmetry** are introduced in relation to the symmetry of the letters of the alphabet.

2. **Design Cards**
   Format - Small groups at desks
   Time - 1 to 2 periods
   Materials
   One set of Design cards per group
   Design Cards Worksheets
   Swirling Mirrors Worksheets

   Students classify 'Design Cards' on the basis of what kinds of symmetry they have: none, number of lines of mirror symmetry, and degree of rotational symmetry.

*3. **Your Own Symmetry Designs**
   Format - Small groups at computers
   Time - 30 minutes per group of students at the computer
   Materials
   Pet computer(s) loaded with Turlteworks program
   Your Own Symmetry Designs Worksheets

   Students create designs on the computer which have various symmetry properties and store some of them on the network. They look at designs other students have stored and categorize them according to their symmety properties.

4. **The Polygon Rosette Family**
   Format - Small groups at desks with computers available for checking
   Time - 1 to 2 periods
   Materials
   Pet computer(s) loaded with Turtleworks program

Polygon Rosette cards
Polyton Rosette Worksheets

Students invent a classification scheme for the 'Polygon Rosette Cards'. which ends up being based on two attributes of the cards: symmetry, and kind of polygon.

5. **Symmetry Families**
Format - Small groups at computers
Time - 30 minutes per group of students at the computer
Materials
Pet computer(s) loaded with Turtleworks program
Symmetry Families Worksheet

Students create their own families of symmetric designs where each member of the family they create has a different symmetry than the other members. They learn to use the 'TO' command of Turtletalk.

Activity 1
Alphabet Symmetry

Format
Whole class discussion plus work in pairs

Time
1 class period

Materials Needed
large cutout letters
overhead projector
turtle tractor
Greek Letters Symmetry Worksheets

Background

Much of the work in the previous unit has had students working with symmetric drawings. It is time to pin some labels on two of the kinds of symmetry they have seen: **mirror symmetry** and **rotational symmetry**.

A design with a **line of mirror symmetry** has a line in it such that if you place a mirror along the line and look into the mirror, the design will look unchanged. The letter '**M**' has mirror symmetry dividing it in half vertically. The '**C**' has a line of mirror symmetry dividing it in half horizontally. The letter '**H**' has two lines of mirror symmetry, one vertical and the other horizontal.

Another way to check for lines of mirror symmetry is to fold the design along the supposed line. If the two halves match up exactly, then the design does indeed have a line of mirror symmetry there.

Rotationally symmetrical patterns can be spun around a certain point and they will match up with themselves. For example, the letter '**S**' has 180 degree rotational symmetry because it can be turned upside down (or 180 degrees around) and look just the same. The letter **X** (if drawn appropriately) has 90 degree rotational symmetry because it will still look the same if turned on its side (or 90 degrees).

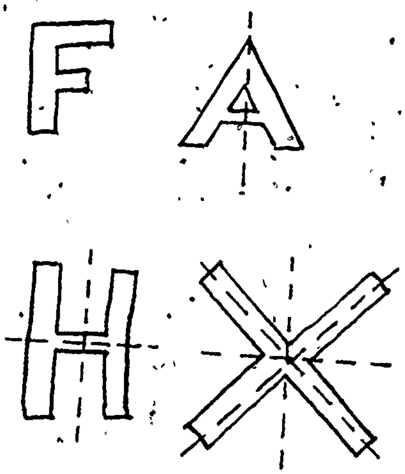
Purpose

- Students should be able to classify the letters of the alphabet according to whether they have **no symmetry, mirror symmetry only, rotational symmetry only, or both rotational and mirror symmetry**.

Teacher Preparation

Make copies of the Greek Letters Symmetry worksheet.

It will be helpful to have some large cutout letters to demonstrate various symmetries. Good

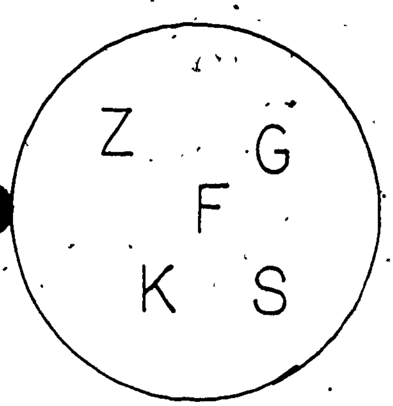


choices would be the letters **F** (because it has no mirror symmetry), **A** (because it has one line of mirror symmetry), **H** (because it has two lines of mirror symmetry), **X** (because it has four lines of mirror symmetry if constructed appropriately, and, likewise, 90 degree rotational symmetry), **S** (because it has 180 degree rotational symmetry), and **O** (made round to have infinite mirror and rotational symmetry).

Also have on hand a turtle-tractor to demonstrate degree of rotational symmetry.

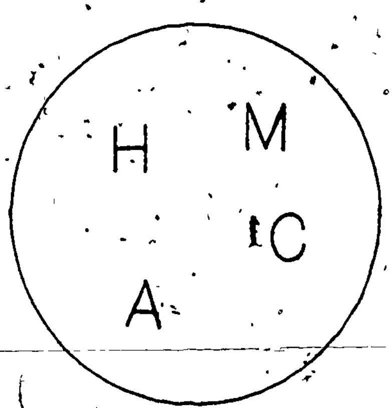
Activity

Mirror Symmetry



On the board (or using an overhead transparency), make two large circles. Tell the students that some of the letters of the alphabet belong in one circle and others belong in the other circle. Their job is to figure out the 'rule' that determines which letters belong where. When they figure out the rule, they are **not** to tell the rule (which would spoil the fun of figuring it out for their classmates), but merely to raise their hand with one finger up to signal to you that they know the rule.

Students suggest letters. In one circle, put letters that have at least one line of mirror symmetry. In the other circle put letters that have no lines of mirror symmetry. Proceed slowly, allowing students plenty of time to ponder the evidence already on the board.



As soon as a student indicates knowing the rule, let him/her take over the placement of letters in the circles. They can stay up until they make a mistake, or until another students indicates knowing the rule.

Some groups will find figuring out the rule extremely difficult. If this seems to be the case for your class, stop part way through and have a discussion about what kinds of ideas they have had. Are the focussing on vowels and consonents, number position in the alphabet, words that begin with those letters? Give them the hint that your rule has only to do with **the shape of the letter.**

Finally, most of the class should have the rule figured out. Several students should explain the rule. The words '**symmetry**', '**mirror symmetry**', and '**line of mirror symmetry**' should be introduced, too. The labels

Has at least one line of mirror symmetry
and
Has no line of mirror symmetry

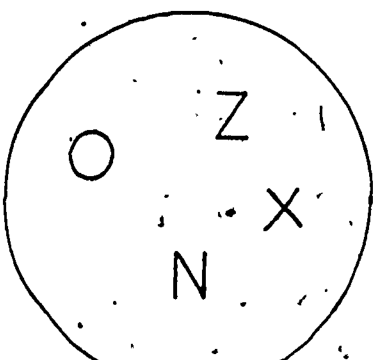
should be placed above the appropriate circles.

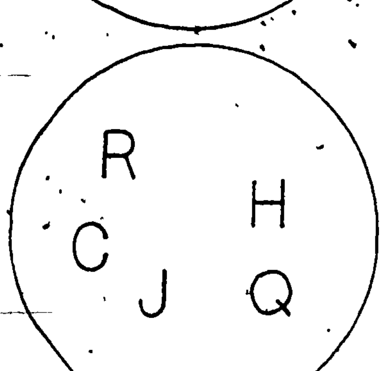
Ask for suggestions about how one might determine whether a letter (or any shape) has a line of mirror symmetry. Discuss

- folding a cut-out of the shape. (Demonstrate this with a letter which has mirror symmetry and one that doesn't. The letter **S** is a good one to use as a counter-example since to some, it will seem to have mirror symmetry.)

- placing a mirror along the supposed line of symmetry and seeing if the reflected image looks like the rest of the shape.

- any other methods they generate.





Look at the letter I which has **two** lines of mirror symmetry. Can they find any other letters with two lines of mirror symmetry? Can they find any letters that have more than two lines of mirror symmetry? How many lines of mirror symmetry does the letter **O** have? (A circle is said to have 'infinitely many line of mirror symmetry.' The letter O, however, is usually not constructed as a perfect circle.)

Rotational Symmetry

Now repeat the rule game from the beginning. This time, one circle will contain letters that have **rotational symmetry** and the other circle will contain letters that have **no rotational symmetry**.

They may explain the rule in terms of 'turning the letter upside-down.' Emphasize that after turning the letter by some degrees, it looks just the same as it did. How many degrees does one turn the letter **H** to have it look just the same? Explain why the letter H has 180 degree rotational symmetry. Examine each of the letters that have some rotational symmetry and label them with how many degrees of rotational symmetry they have. (The letter O is said to have 'infinite rotational symmetry.')

You can use an overhead and a turtle-tractor to demonstrate the turning operation. Make a large letter on a transparency. Place the turtle down on top of it. Turn the turtle and the letter through the appropriate angle. Lift up the turtle and verify that the letter looks just as it did before the turn.

Greek Letters Worksheet

Working in pairs, the students can practice recognizing different symmetries using the small Greek letters which are pictured on the Greek letters worksheet.

Homework

An appropriate homework activity would involve cutting out pictures or designs with different kinds of symmetry and bringing them to school to post on the bulletin board.

Activity 2
Design Cards

Format
Small groups at desks

Time
1 to 2 class periods

Materials Needed
- one set of design cards per group
- Design Cards worksheets
- Swirling Mirrors worksheets

Background

Classification is a basic skill used constantly in mathematics and the sciences. When you (and your students) first look at the set of 'Design Cards,' you will see a bewildering variety of designs. And yet, as you look more closely, you will see that there are attributes that the cards have which allows you to group some of them together. Nearly all the designs have some kind of symmetry. Some of them have mirror symmetry. Some have rotational symmetry. Some have both mirror and rotational symmetry. Of those that have mirror symmetry, there are designs which have only one line of mirror symmetry, others which have two lines, three lines, and so on. All those designs which have two or more lines of mirror symmetry also have rotational symmetry. Those which have two lines of mirror symmetry have 180 degree rotational symmetry, those which have three lines of mirror symmetry have 120 degree rotational symmetry, four lines implies 90 degree rotational symmetry. There is a relationship between the number of lines of symmetry and the degree of rotational symmetry. From chaos emerges startling and beautiful order.

Purpose

- Given a design, students should be able to show any lines of mirror symmetry the design might have
- Given a design, students should be able to state the degree of rotational symmetry the design has
- Given a set of designs, students should be able to classify the designs according to their symmetry properties
- Being told how many lines of mirror symmetry (passing through a common point) a design has, students should be able to predict the degree of rotational symmetry of the design.

Teacher Preparation

The first time you do this activity, you will need to make a classroom set of 'Design Cards.' These cards are printed on 8 1/2 x 11 inch cardboard and should be cut into individual cards.

You will also need copies for students of the two worksheets 'Design Cards Worksheet,' and the 'Swirling Mirrors Worksheet.'

Do the activity yourself, especially with an eye to understanding how they might appear to students the first time. Very often, students will first sort the cards into two, three, or four groups. See if you can figure out what these might be (more about this later).

Activity

Introduction

You should not need to say much after having had the class form into groups and passed out worksheets and Design Cards. Most importantly you do **not** want to suggest initially to your students that they classify the cards according to symmetry (although it may become necessary to give them this hint later). The idea is for them to decide **themselves** that the most reasonable scheme is based on symmetry.

While Students Are Working

There are several problems that may occur after groups begin to work.

First, some groups will choose completely different attributes for classification besides symmetry. That they do this is not a terrible thing, nor does it necessarily say anything about how well they understood the previous day's lesson; nevertheless, you want to steer them back to looking at symmetry ideas. To do this, you should find out if they really have a solid criterion for distinguishing their different groups. Probably they don't. For example, putting small designs in one pile and large designs in another pile doesn't really work, because there is no clear boundary between small and large.) You can suggest that they look for other factors that will be easier to apply to the cards. If their criterion is quite applicable (e.g. lifelike designs in one pile and abstract designs in another), complement them on their idea; and then suggest to them that they see if they can use the symmetry ideas discussed the previous day in classifying the cards.

Second, many groups will sort the cards into just two (or possibly three) piles: designs that have no symmetry, and those that do have symmetry. When this happens, you can suggest that they take the pile of cards with symmetry and further break it down into smaller piles. Eventually each pile should contain cards which have exactly the same symmetry properties (e.g. 90 degree rotational symmetry and four lines of mirror symmetry passing through the center).

Third, some groups won't get the idea of classifying. They just won't know where to start. You can help them by just reiterating that their task is to 'put cards in groups according to which ones belong together.' Perhaps give them an example with some cards that have mirror symmetry and some that don't. Ask them which look more similar.

Finally, some groups will have trouble understanding what it means to put a label on a group. They need to know that labels are a way of communicating to other people the method they used to sort the cards. You should check their labels to see if they would be clear to someone examining their work.

As groups finish, you should ask them to copy the labels they come up with on the board. These will form the basis of a discussion about their classification schemes.

Summary Discussion

For your reference, here are the eight symmetry groups into which the design cards fall:

1) No symmetry
2) 1 line of mirror symmetry
3) 180 degree rotational symmetry and no lines of mirror symmetry
4) 180 degree rotational symmetry and two lines of mirror symmetry

5) 120 degree rotational symmetry and no lines of mirror symmetry
6) 120 degree rotational symmetry and three lines of mirror symmetry
7) 90 degree rotational symmetry and no lines of mirror symmetry
8) 90 degree rotational symmetry and three lines of mirror symmetry

Of course the actual words that the students use to describe the groups will be quite different.

Referring to the lists of labels the students have put on the board, you ask questions of the whole class like the following:

- Are these two classification schemes the same? If they are different, in what way are they different?

- Where would this card go in these scheme?

- Can you think of a design which would not belong anywhere in this scheme?

- Can you think of a design which would not belong anywhere in any of these schemes?

- Where would a square go?

- Where would a 12-sided regular polygon go?

- Where would a circle go?

- Which of the ways of labelling do you like the best and why?

Swirling Mirrors

The Swirling Mirrors Worksheet is designed to provide students with a chance to tie together some insights they might be having about the relationship between the number of lines of mirror symmetry and the degree of rotational symmetry. This relationship (which they are to discover) is that

> **Whenever a certain number (call this number N) of lines of mirror symmetry pass through a common point, the design will have**
>
> **360/N**
>
> **degrees of rotational symmetry.**

Working in their groups, they should go through the worksheet with design cards in hand.

When they are through, discuss with the whole class the relationships the found.

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Activity 3
Your Own Symmetry Designs

Format:
Small groups at computers

Time
30 minutes per group of students at the computers

Materials Needed
Pet computers loaded with Turtleworks

Your Own Symmetry Designs worksheets

Background

Having been introduced to the basic concepts of mirror symmetry and rotational symmetry, students need a chance to "play around" with creating their own designs. The Turtleworks computer program creates an ideal environment for this creative play because symmetrical designs may be created very quickly and in a very straight-forward manner.

Purpose

- Students should be able to create mirror symmetrical and rotationally symmetrical patterns using Turtletalk on the computer. They should be able to look at other students' designs and classify them according to their symmetry.

Teacher Preparation

Try creating symmetrical designs yourself on the computer. Store one of your designs on the network. Look at some of the designs which are already stored there. Have the Turtleworks program loaded before class. Have copies of the worksheets available. Have regular classwork for students who are not taking their turns on the computer.

Activity

Divide students into groups of three and set up a schedule for computer use. When groups begin their work at the computer, they will need to have individual copies of the worksheet.

The first page gives them practice with a Turtleworks program that makes patterns with 72 degree rotational symmetry. Then they are asked to make patterns of their own. Part of the fun of the activity is saving one's designs. If you can connect to the MNCP Network, students can save their designs on the Network and look at the designs that other students have stored there. Otherwise, you can create a class library of designs stored on tape. In any event, students will need some help using the NETNAMES, NETSAVE, NETLOAD, TAPESAVE, and TAPELOAD commands.

Activity 4
The Polygon Rosette Family

Format
Small groups at desks with computers available for checking

Time
1 to 2 class periods

Materials Needed
Pet computer(s) loaded with Turtleworks
Polygon Rosette cards
Polygon Rosette Family worksheets

Background

The Polygon Rosette family at first presents a bewildering variety of designs. Quickly, however, the variety sorts itself out. One finds, in the end, that there are only two important attributes that distinguish the members of the family: **number of sides of the polygon**, and **number of polygons**. The cards give students an important medium for creating their own classification scheme which, with your guidance, will converge to a two-dimensional array where the rows differ in one of the attributes and the columns differ in the other.

Purpose

- Students should be able to classify the members of the Polygon Rosette family according to number of sides of the polygon and number of polygons. They should be able to produce a Turtletalk program to draw any member of the family.

Teacher Preparation

The first time you do this activity, you will need to make a classroom set of Polygon Rosette cards. The masters to these are included in the unit. The beginning set is on white paper while the cards to make an extended set are on blue paper. Run these off on card stock (white and blue respectively) and then cut them up into individual cards on a paper cutter. To make four sets of cards requires about 1 hour of work with a paper cutter. You also need to have copies of the worksheet on hand.

Activity

Introduction

Students should form into groups and be given a set of cards and worksheets.

The Polygon Rosette cards come in two parts. The 48 white cards contain the simplest patterns. Students should start with these alone. Groups that

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have a good grasp of what is going on can be given the 42 blue cards to add (as opposed to starting over) to their emerging classification scheme.

While They Are Working

Students work in groups and must have in front of them a large enough flat area to lay out the cards. The first part will be the hardest for them. They will probably flounder around for quite a while before beginning to settle in on a reasonable classification scheme for the cards. Make sure they write out a description of what they have done on their paper. You can look over their shoulders and ask questions like "Why did you put that card there?" or "Is there another place this whole row could go?"

Summary Discussion

You will probably want to have the whole class discuss their experience with the cards. Some discussion questions include:

- What systems for placing the cards did you develop? (Show on the board, to serve as a focus for further discussion.)

- What cards were hardest to place? (Usually the six hexagons is hard to place, as are the 'single line' cards.)

- What did you decide the 'single line' cards were, and where did you put them?

- What was the same about the Turtletalk programs that produced all the cards?

Activity 5
Symmetry Families

Format
Small groups at computer(s)

Time
30 minutes per groups of students at the computer

Materials Needed
Pet computer(s) loaded with Turtleworks
Symmetry Families worksheets

Background

In many of the activities of both this unit and the Turtle Geometry unit, students have been exposed to 'families' of designs. The Regular Polygon Family was the first of these. This family is neatly defined by the Turtletalk program

RPT n (GO 10 TURN 360/n)

where n was the number of sides of the regular polygon. The essence of a family is that all the members are 'the same' in some way, and that they all differ in some 'patterned' way. The regular polygons are all closed, straight-sided figures with all equal sides and all equal angles. They differ from each other in the number of sides they have.

Families like these play important roles both in mathematics and in the sciences. To take but one example, the elements of the periodic table form a large family where all members are atoms with electrons around a nucleus. Members differ in the number of electrons they have.

We have, in Turtletalk, an excellent opportunity for students to create **their own** families and study their properties. They will do this by defining a basic shape, giving it a name, and then putting multiple copies of this basic shape together to form more complex shapes. These complex shapes will be the elements of the students' families. Because we are particularly interested in symmetry, students will be asked to characterize the members of their families according to symmetry properties.

Purpose

- Students should be able to generate a family of turtle designs and characterize each member of the family according to its symmetry properties.

Teacher Preparation

There is a new Turtletalk command involved in this work with which you will need to be familiar. It is the TO command. With it, we can define new words in the Turtletalk language. Go through the worksheet yourself before class and experience what it is like to make a symmetry family.

Have copies of the Symmetry Families worksheet made and load up the computers with Turtletalk before class begins. Bring one computer to the front of the class where most students will be able to see it for a demonstration.

Prepare regular classwork for students who are not working on the computers.

Activity

Introduction

Students will need familiarization with the new command, TO. Explain that it is possible to teach the turtles new vocabulary. Write on the board (or overhead) the following:

```
CLEAR

TO BEND
GO 10
TURN 135
GO 10
END

CLEAR
BEND
```

Ask students what they think will be the effect of typing all this in to the computer. Then try it.

Ask what they think the following will produce:

```
CLEAR
MULT 3
BEND
```

When they try it, ask what they think is going on. They should realize that **all** three of the turtles perform the BEND command.

Finally, ask what they think

CLEAR
BEND 3

will do. Before trying it on the computer, go through it with them on the board.

While Students Are Working

The first part of the activity involves learning to use the TO command. Then they are to use it to teach the turtles a new command of their own. Finally, they use the new command to create designs of various symmetries and make a family of designs. There is a great deal of room for creativity.

Students may define commands that are either too simple or too complex for good families. As you notice this happening, suggest that they go back and define another new command to use instead of the one they first try.

Although the worksheet does not suggest this, you may wish students to store their new commands and a representative member of their family on either the MNCP Network or on tape. They may wish to browse other students' designs and see how they were done.

Summary Discussion

The class discussion should center around groups displaying the various new commands they defined and explaining how they used these new commands to create symmetry families.

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Appendix

Turtle Symmetry

Worksheets

Name________________________

Other Group Members________________________

Date________________________

Greek Letters Symmetry Worksheet

Ancient Greeks used a different alphabet than the one we use now (the Roman alphabet). Below are listed the small letters in their alphabet with the names underneath.

α	β	γ	δ	ϵ	ζ
alpha	beta	gamma	delta	epsilon	zeta
η	θ	ι	κ	λ	μ
eta	theta	iota	kappa	lambda	mu
ν	ξ	ο	π	ρ	σ
nu	xi	omicron	pi	rho	sigma
τ	υ	ϕ	χ	ψ	ω
tau	upsilon	phi	chi	psi	omega

1. Which of the Greek letters have **no** symmetry?

2. Which of the Greek letters have **one** line of mirror symmetry?

Greek Letters Symmetry Worksheet

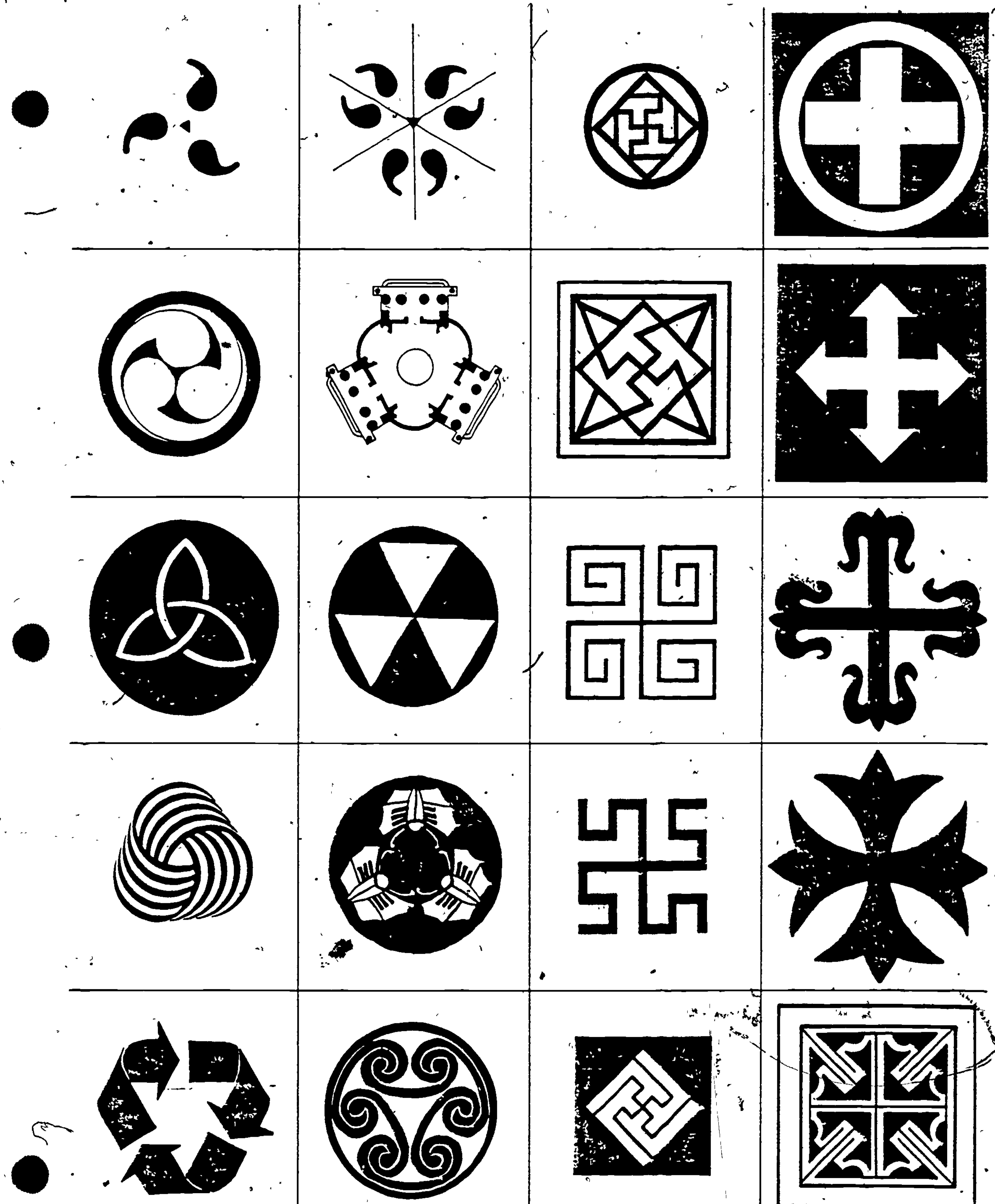
3. Which of the Greek letters have **more than one line** of mirror symmetry?

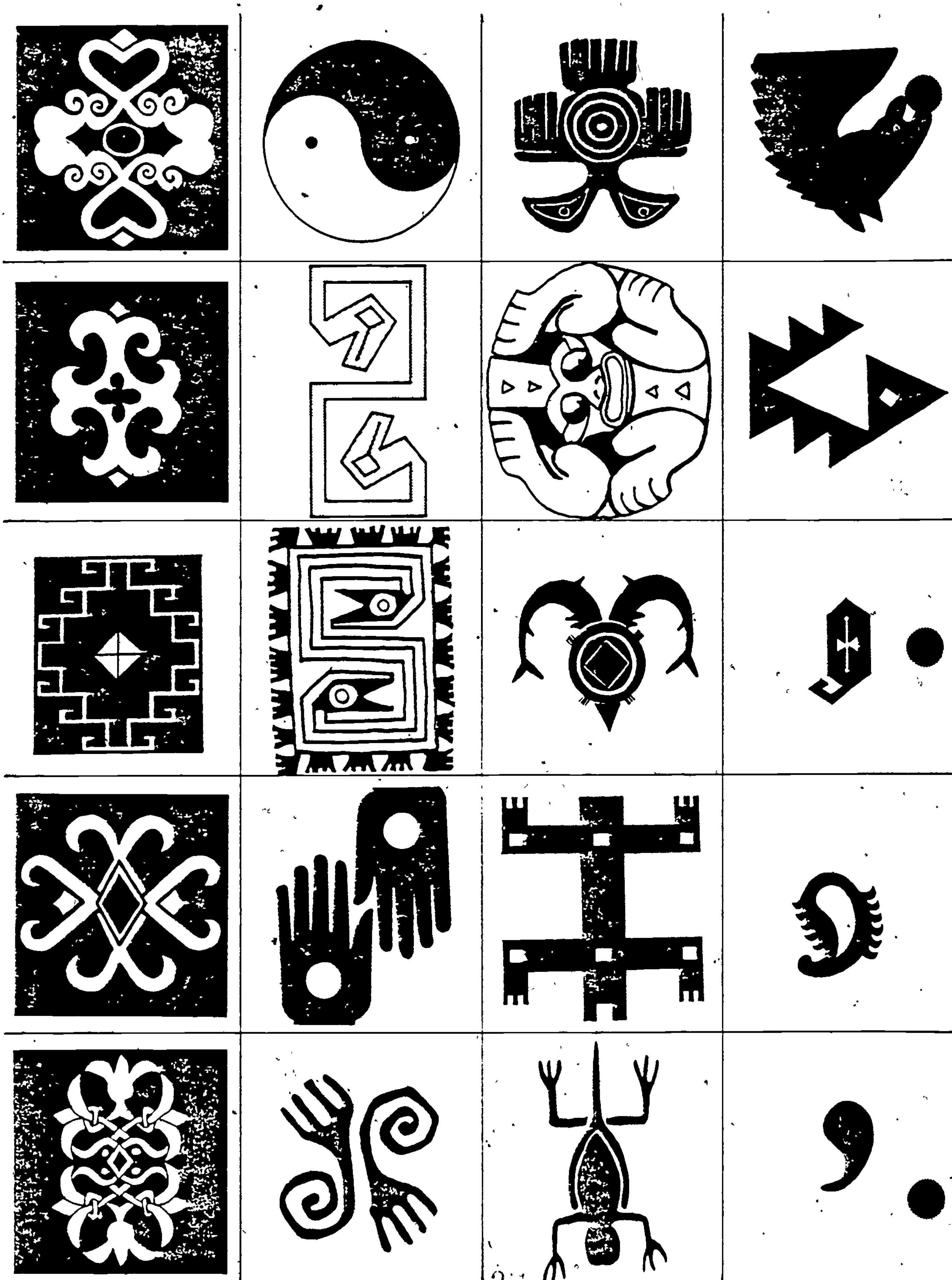
4. Which of the Greek letters have **rotational** symmetry?

5. Which of the Greek letters have **both rotational and** mirror symmetry?

6. Which of the Greek letters have rotational **but not** mirror symmetry?

25





Name________________________

Other Group Members______________________

Date________________________

Design Cards Worksheet

Materials: You need a set of 'Design Cards' and fairly large, flat area (the floor or a table will do fine). You will also need quarter-sheets of paper for making labels.

1. Spread the cards out and look through them with the other members of your group.

2. Your task is to find a way to group those cards together which 'belong' together. You will have to decide what it means to 'belong' together. Lay the cards out on the table so that all the cards are visible and the cards which belong together in a group are close to each other. (You will find that when you are done there will be more than four groups and less than ten groups.)

3. Make a label for each group on a quarter-sheet of paper. Decide together what the labels should say. Your labels should make it clear which designs belong in each group so that another person could use just your labels to group the design cards.

   Record your labels here:

Name________________________

Other Group Members____________________________

Date________________________

Swirling Mirrors Worksheet

Materials: You need a set of 'Design Cards' and your records from the 'Design Cards' Worksheet.'

1. In your work with the Design Cards, you probably realized that there were lots of kinds of symmetry that could have been present and were not. For example, there were no designs that had five lines of mirror symmetry and 72 degree rotational symmetry.

   Name another possible kind of symmetry that none of the cards have.

   Sketch a design that has that kind of symmetry.

2. Two of the following kinds of symmetry are **impossible!** Which two are they?

   (a) two lines of mirror symmetry and 90 degree rotational symmetry

   (b) six lines of mirror symmetry and 60 degree rotational symmetry

   (c) three lines of mirror symmetry and no rotational symmetry

Swirling Mirrors Worksheet

3. **The Relationship Between Mirror Symmetry and Rotational Symmetry**
Fill out the chart below with all the kinds of symmetry you have met so far. For example, the first line has been filled out to represent designs with 2 lines of mirror symmetry and 180 degree rotational symmetry.

Number of Lines of Mirror Symmetry	Degree of Rotational Symmetry
2	180

Look only at the lines in the table that do have rotational symmetry. What relationship do you see between the left column and the right column?

Describe the relationship here.

30

Name_______________

Other Group Members_______________

Date_______________

Your Own Symmetry Designs Worksheet

Materials: You need to be working in front of a PET computer which is running the Turtleworks program.

1. Try the program below. Make a sketch of the design in the space provided.

```
MULT 5
GO 10
TURN 45
GO 5
```

   What symmetry does this design have?

2. Here is another program. Try it on the computer and sketch the result here.

```
MULT 5
GO 10
TURN 45
GO 5
TURN 119
GO 5
TURN 45
GO 10
```

   What symmetry does this design have?

Your Own Symmetry Designs Worksheet

3. Now try creating your own designs. Make one that has **only** rotational symmetry and one that has **both** rotational symmetry **and** mirror symmetry. When you get ones you like, record the program here.

ONLY ROTATIONAL SYMMETRY	BOTH ROTATIONAL AND MIRROR SYMMETRY

4. **Saving your designs**
If your PET can be connected to the MNCP Network, save your designs there using the NETSAVE command. Otherwise, save them on a cassette tape using the TAPESAVE command.

Write the name of your program here ________________

5. **Looking at other designs**
Load someone else's design into your PET and decide what symmetry it has. If you can connect to the MNCP Network you can use the NETNAMES command to see the names of the designs stored on the network. Then type NETLOAD followed by the name of the chosen design. Otherwise use the TAPELOAD command to load a classmate's design from tape.

Name of the program I looked at: ________________

Comments on the program (including what kind of symmetry it had:

32

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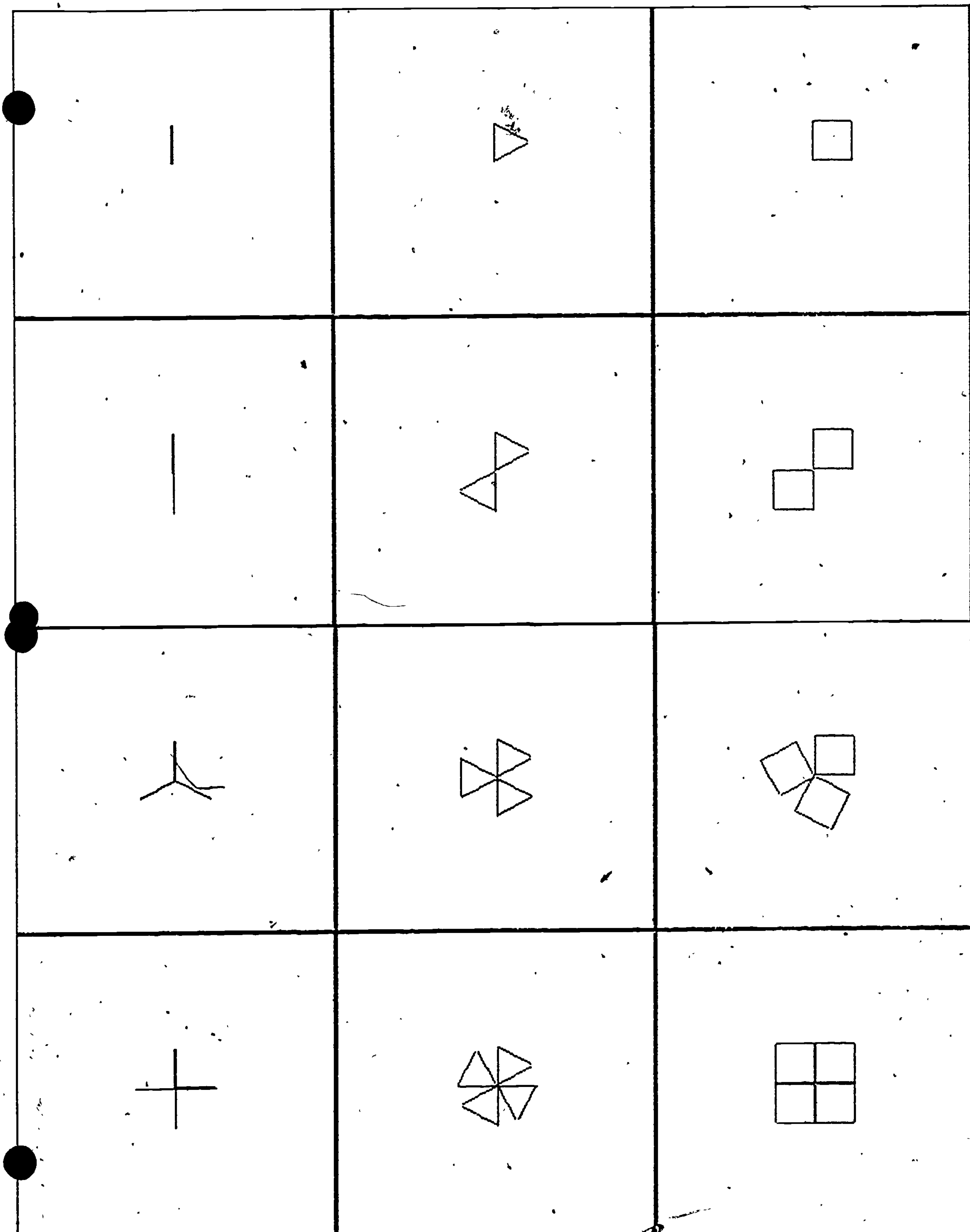

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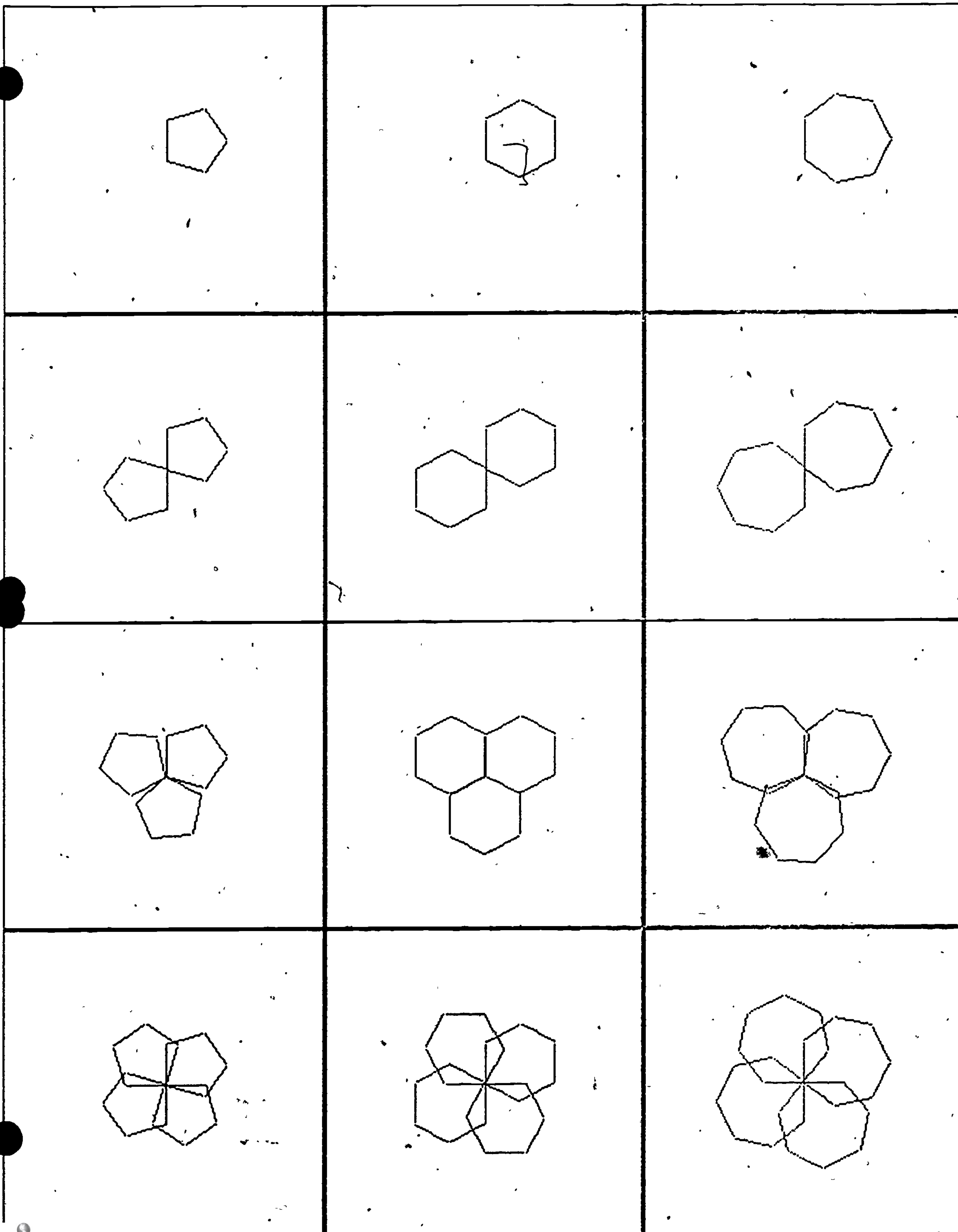

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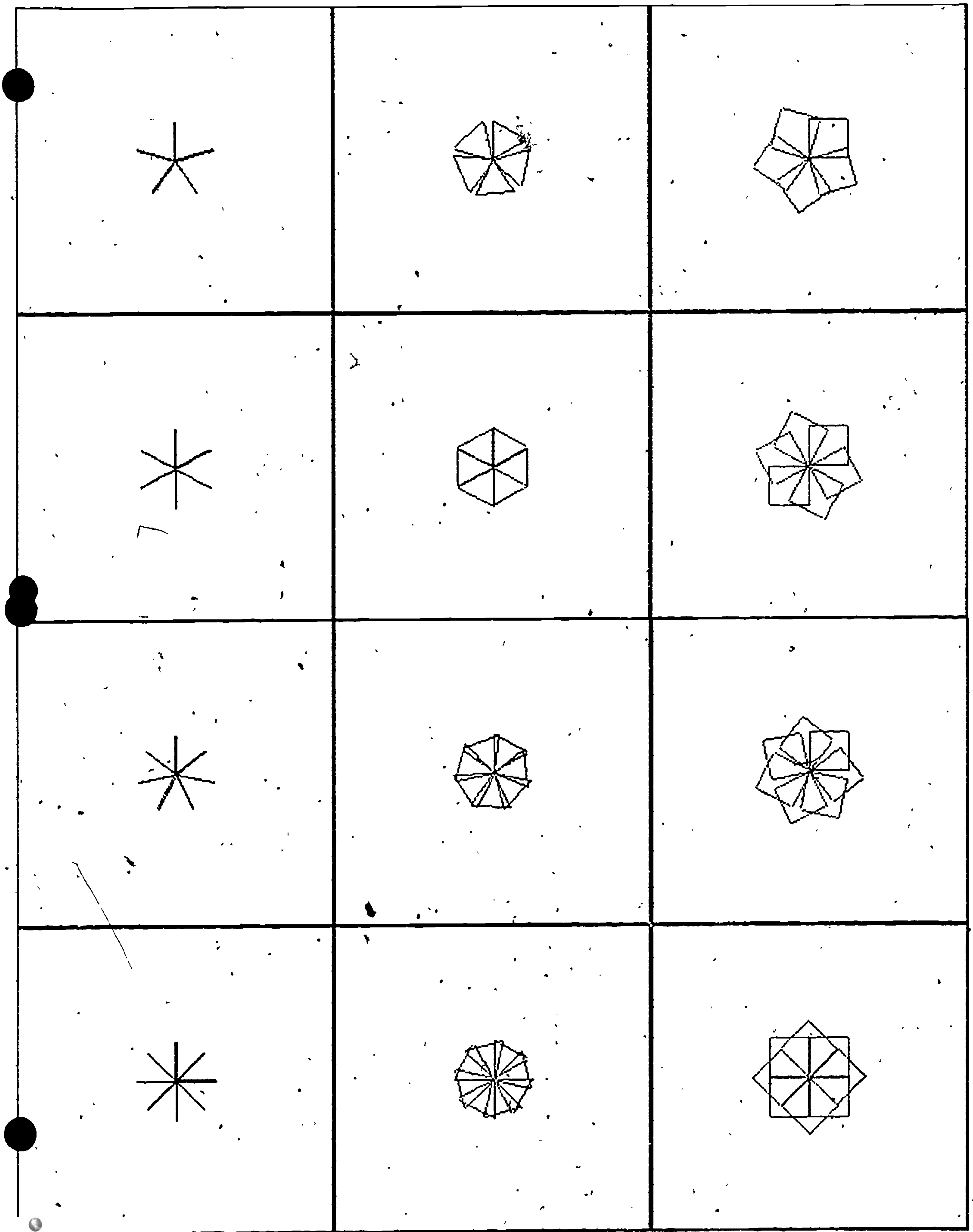

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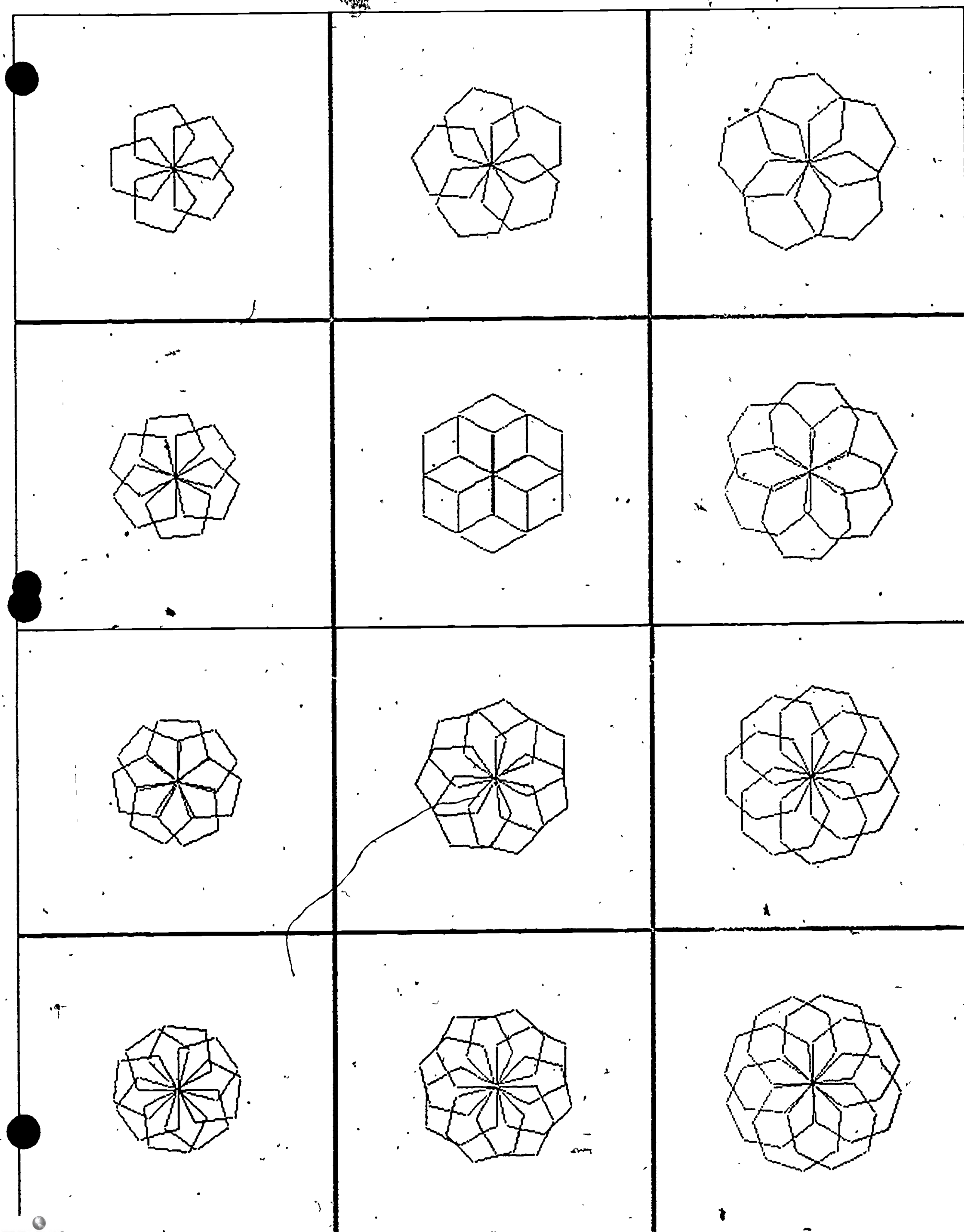

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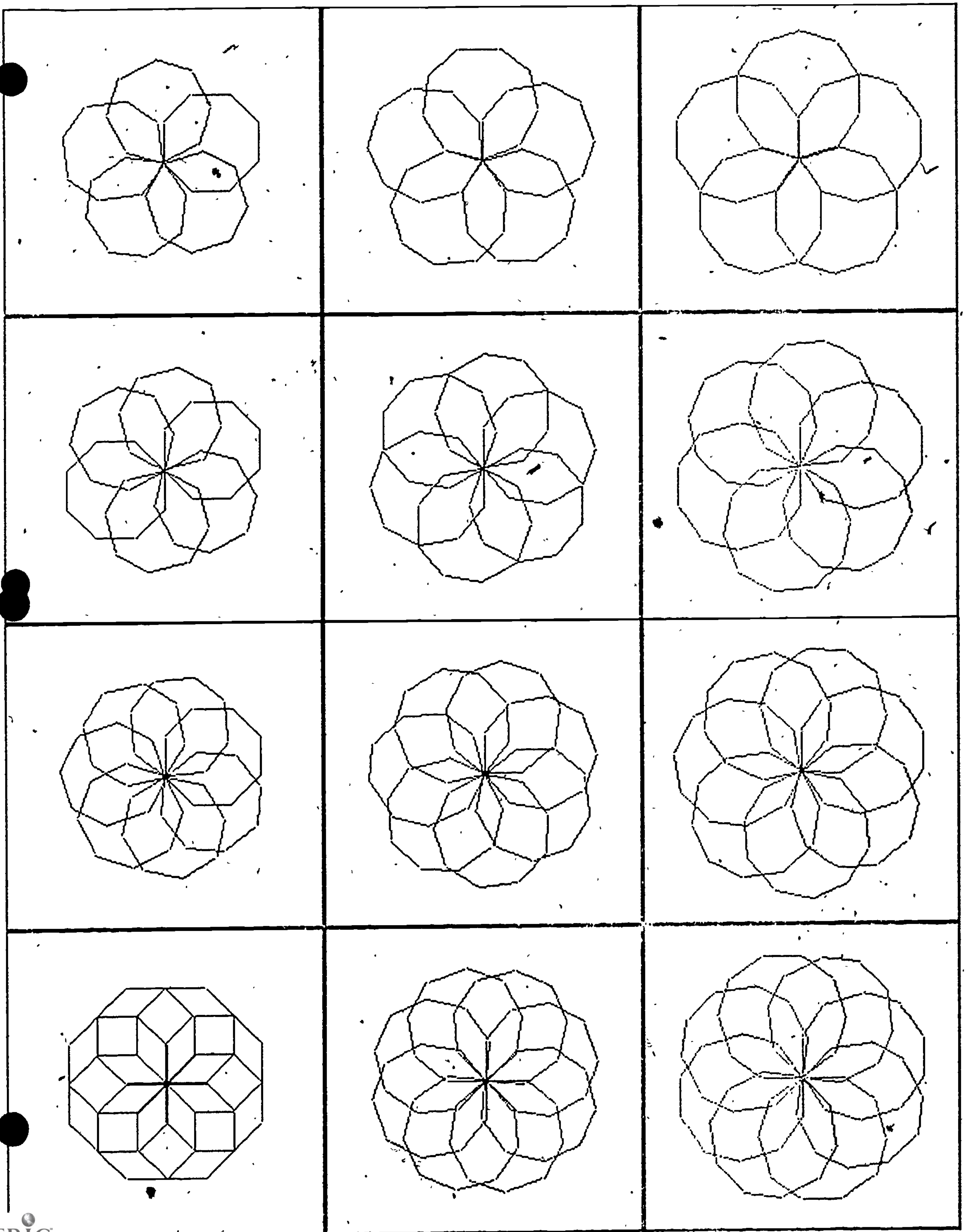

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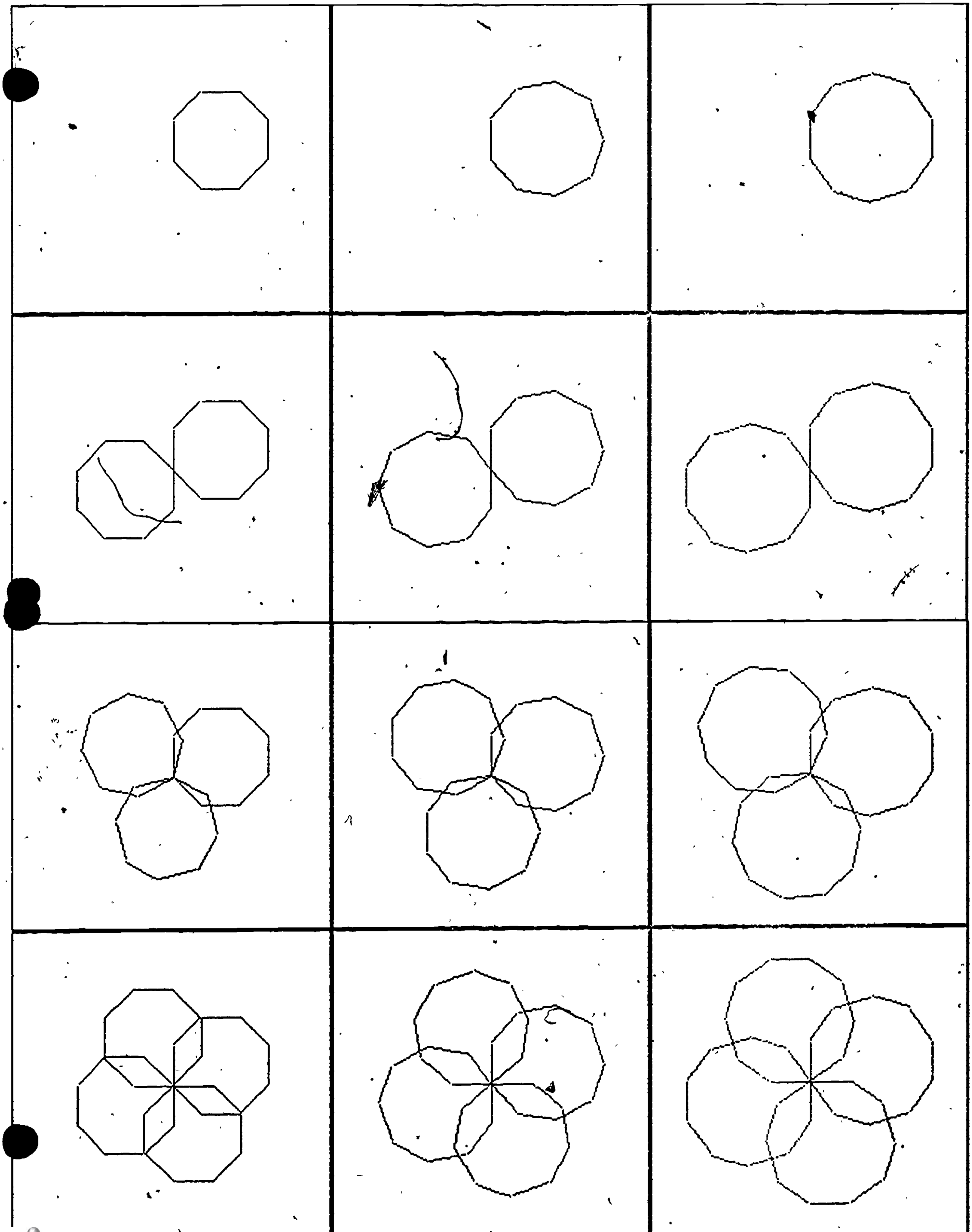

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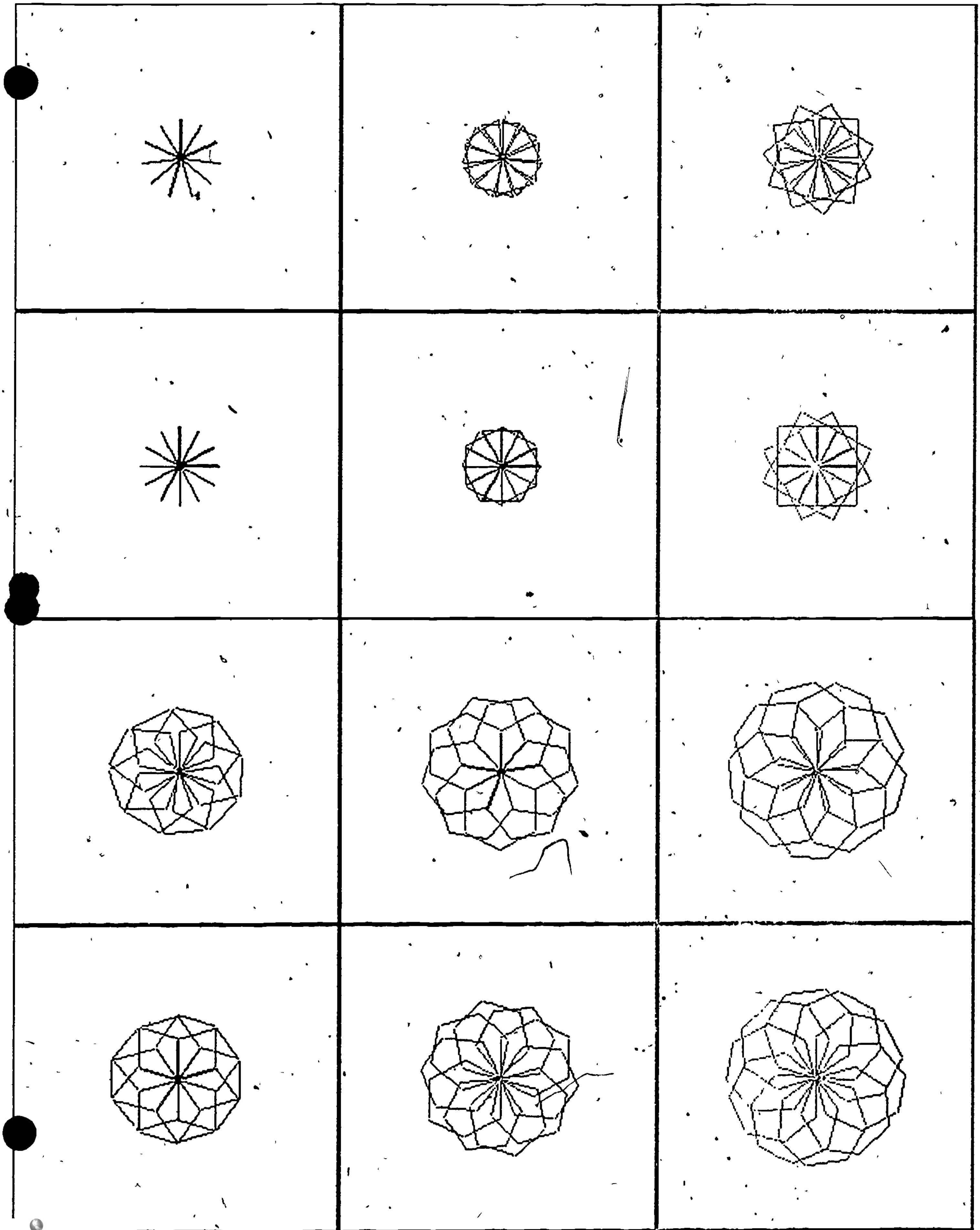

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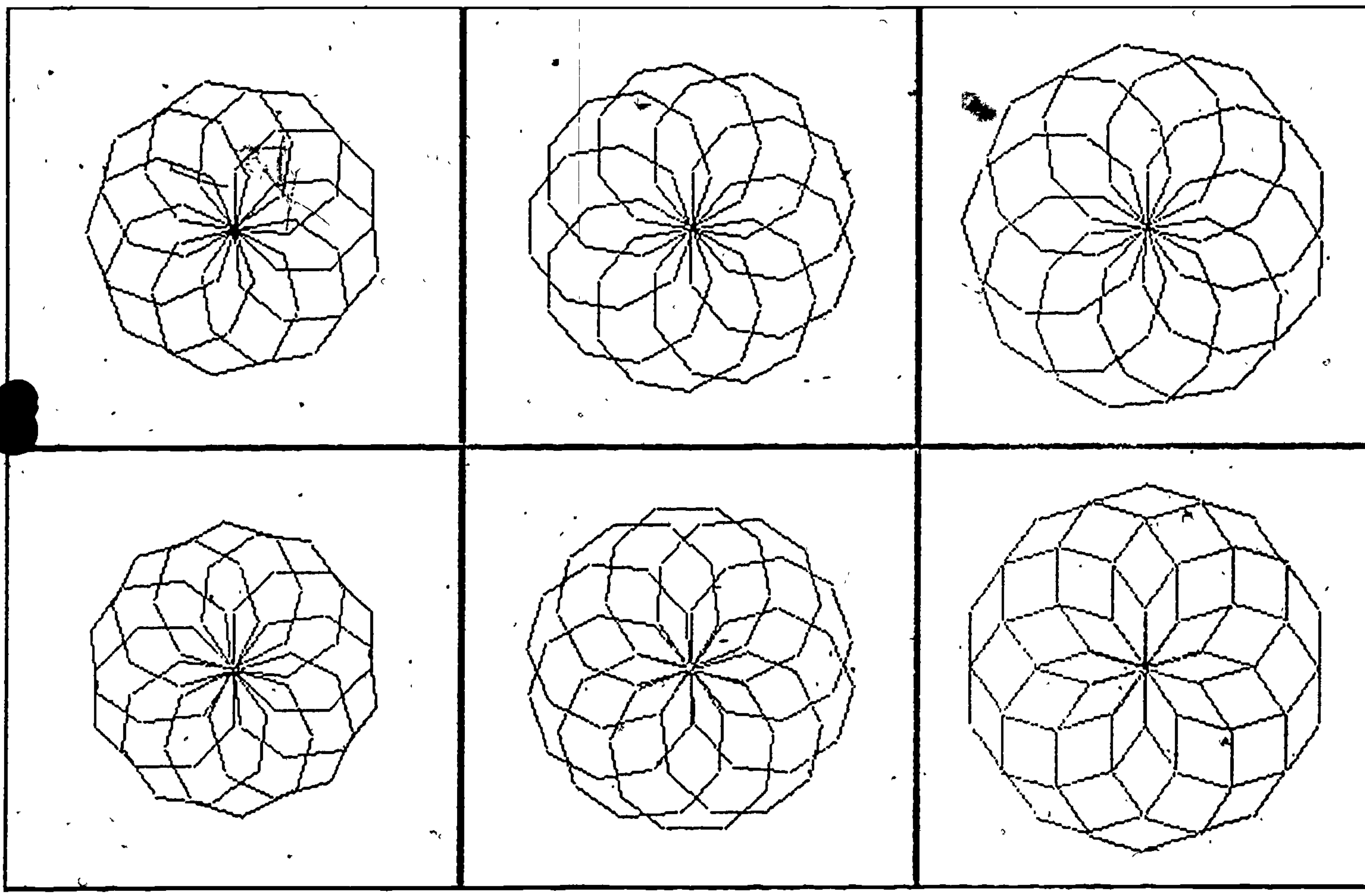
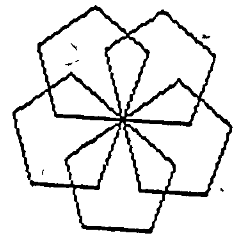


Name ____________________

Other Group Members ____________________

Date ____________________



The Polygon Rosette Family Worksheet

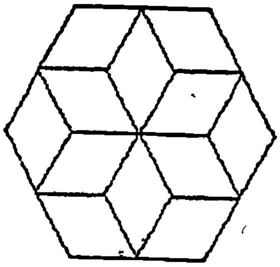
Materials: You need a set of 'Polygon Rosette cards' and a fairly large flat area (the floor or a table will do fine).

1. Spread out the cards so you can see most of them. Your task is to 'organize' the cards in a system which makes it easy to see the similarities and differences between different cards. When you are done, all the cards will be visible and there will be a kind of pattern to the way the cards are arranged. Describe your system here.

2. If you have done a good job of laying out the cards, it should be easy for you to find each of the patterns listed below. See if you can. You may find you want to revise your arrangement. For each card, describe where it lies in your pattern.

Description of card	Place in pattern
3 squares	
5 triangles	
3 hexagons (six sides)	
4 heptagons (seven sides)	
8 heptagons	

Describe in writing here why you decided to put the cards with single lines where you did.



Polygon Rosette Family Worksheet

3. These cards were produced by a computer which understands Turtletalk. You should be able to write a Turtletalk program to produce any of the cards. Pick two cards and figure out their programs.

Description of card	Turtletalk Program
1.	
2.	

4. Each of the two Turtletalk programs below will draw one of the cards. Describe the appropriate card in the right-hand column.

Program	Description of card
1. MULT 7 RPT 4 (GO 10 TURN 90)	
2. MULT 6 RPT 5 (GO 10 TURN 72)	

5. Some of Polygon Rosettes belong together in a special way. For example, look at the **four squares**. See how they come together to make a single larger square. What other card does the same kind of thing?

If you have the extended set of Polygon Rosette cards (the extra cards are colored blue), find two other cards in which the small shapes also come together to make a single larger shape with the same outline. What are they?

Find **another special family** of Polygon Rosette cards. Describe that family here.

Name ____________________

Other Group Members ____________________

Date ____________________

Symmetry Families Worksheet

Materials: You need to be working in front of a computer loaded with the Turtleworks program.

1. **Making a Design Out of Parts**
You can teach the turtles new words. Here is an example. Type in each line, hitting <return> at the end of each line.

```
TO POP
GO 8 MULT 3
TURN 45 GO 4
END
```

You have just told the turtles what **POP** means and you can see what it means on the screen. From now on you can use the command **POP** just like any other Turtletalk command.

Try this:

```
CLEAR
MULT 3
POP
```

and this:

```
CLEAR
POP 3
```

Play around with **POP** a bit before going on.

2. **Make Your Own**
As a group, try teaching the turtles a new command of your own. The first thing you type in is

```
TO
```

followed by whatever you choose to name the command.

The last line of the definition of the command must be

```
END
```

Symmetry Families Worksheet

Write the name and definition of your command here:

TO ________________

END

3. **Using Your New Command**
See what happens when you

CLEAR
MULT 2

and then issue your command.

Does the design created have rotational symmetry?
______If so, what degree? ________

Does the design created have mirror symmetry? ______
If so, how many lines of mirror symmetry? ________

Find a way to use your command to create a design with 60 degree rotational symmetry. Write how you did it here:

Experiment with different symmetries you can get using your new command. Record the program that produced the most pleasing effect here:

Symmetry Families Worksheet

4. **Your Own Symmetry Family**
   With your new command, you can create 'families' of designs. Here are three designs which are members of a family. Try them.

CLEAR	CLEAR	CLEAR
MULT 2	MULT 3	MULT 4
POP	POP	POP

Show how you would use **your** new command to create a 'family' of designs, each of which has a different symmetry from all the others. Describe your family here.

Show the basic program that you used to create your family. Indicate which parts of the program change to get new members of the family.